MATH 101 CHEAT SHEET

This packet is not meant to go over everything that we have done in the course but rather to summarize a lot of information. In fact, you'll notice that I did not mention **composition and inverses of functions** here, yet I would highly recommend studying these. Please make sure to do the practice finals on canvas and in the workbook. **It is impossible to learn/understand math without doing math; this includes writing all your work down!**

1. Functions

(1) A **function** is a relationship between two variable in which every input has exactly one output.

(2) A function is generally written as $f(x)$, where x is our input variable and $f(x)$ is our output variable. As an example, we write $f(3) = 5$ to say that "when we plug in 3 into the function $f(x)$, the output is 5".

(3) The **domain** of a function is the set of values we are allowed to plug into the function. Think: the x-values we can plug in.

(4) The **range** of a function is the set of values we can get out of the function. Think: the y-values we can get out.

(5) The **average rate of a function** $f(x)$ **on the interval** $[a, b]$ is given by

$$\frac{f(b) - f(a)}{b - a}.$$

2. Types of Functions

(1) A **linear function** is a function of the form $y = mx + b$, where $m = \frac{\text{change in } y}{\text{change in } x}$ is the slope (average rate of change) of the line and $(0, b)$ is the $y-$intercept.
(2) An **exponential function** is a function of the form $y = ab^t$, where a is called the **initial value** and b is called the **growth factor**. Often we consider $r = 1 - b$, which we call the **growth rate**.
(3) A **quadratic function** is a function of the form $y = ax^2 + bx + c$.

3. Linear Functions

(1) **Point-Slope Form:** Given a point (x_0, y_0) on a line with slope m, the equation of the line is $y = m(x - x_0) + y_0$.
(2) **Slope-intercept Form** A line with slope m and y-intercept $(0, b)$ is given by the equation $y = mx+b$.

4. Exponential Functions/ Compound Growth

(1) Here are some properties of exponents:
  (a) $a^x \cdot a^y = a^{x+y}$.
  (b) $\frac{a^x}{a^y} = a^{x-y}$.
  (c) $(a^x)^y = a^{xy}$.
  (d) $a^0 = 1$.
(2) The amount accumulated in an account bearing interest **compounded** n**-times per year** is given by

$$A(t) = P_0 \left(1 + \frac{r}{n}\right)^{nt},$$

noindent where P_0 is the principal (initial) value, r is the nominal interest rate, and t is the number of years.

(3) **CAUTION!** The growth factor of $A(t) = P_0\left(1+\frac{r}{n}\right)^{nt}$ is $\left(1+\frac{r}{n}\right)^{n}$ **not** $\left(1+\frac{r}{n}\right)$

(4) The effective annual rate of an account bearing interest r and compouded n-times per year is given by

$$\left(1+\frac{r}{n}\right)^{n}-1$$

(5) The amount accumulated in an account bearing interest **compounded continuously per year** is given by

$$C(t) = P_0 e^{rt},$$

where P_0 is the principal (initial) value, r is the continuous interest rate, and t is the number of years.

5. Logarithims

(1) $\log_b(t) = y$ is the same thing as saying "the power of b gives me t is y".

(2) We can translate information about logs into information about exponents and vice versa: $\log_b(t) = y$ if and only if $b^t = y$.

Example: $\log_2(8) = 3$ says that $2^3 = 8$.

Example: $3^4 = 91$ says that $\log_3(91) = 4$.

(3) Here are some properties of logs:

(a) $\log_b(xy) = \log_b(x) + \log_b(y)$.
(b) $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$.
(c) $\log_b(x^k) = k\log_b(x)$.
(d) $\log_b(1) = 0$
(e) $\log_b(b) = 1$

(4) Here is how **exponentials and logs** undo each other:

(a) $b^{\log_b(x)} = x$.
(b) $\log_b(b^t) = t$.

The above are very useful for solving exponential and logarithmic equations!

6. Function Transformations

(1) The graph of $f(x-h)$ is the graph of $f(x)$ shifted $|h|$ units **to the left if h is negative** or $|h|$ units **to the right if h is positive**.
(2) The graph of $f(x)+k$ is the graph of $f(x)$ shifted $|k|$ units **up if k is positive** or $|k|$ units **down if k is negative**.
(3) Let $0 < a \leqslant 1$ the graph of $f(ax)$ is the graph of $f(x)$ **horizontally stretched by a factor of** a.
(4) Let $a \geqslant 1$ the graph of $f(ax)$ is the graph of **horizontally compressed by a factor of** a.
(5) Let $0 < a \leqslant 1$ the graph of $af(x)$ is the graph of $f(x)$ **vertically compressed by a factor of** a.
(6) Let $a \geqslant 1$ the graph of $f(ax)$ is the graph of **vertically stretched by a factor of** a.
(7) The graph of $f(-x)$ is the graph of $f(x)$ reflected across the y-axis.
(8) The graph of $-f(x)$ is the graph of $f(x)$ reflected across the x-axis.
(9) When we want to combine transformations, we need to get our function into the form $af(b(x-h)+k$. In which case, we do the following transformations in this order:
   (a) Reflection across y-axis, horizontal compression/stretching by a factor of $|b|$.
   (b) Horizontal shift by $|h|$.
   (c) Reflection across x-axis, vertical compression/stretching by a factor of $|a|$.

(d) Vertical shift by $|k|$.

7. Quadratic Functions

(1) A quadratic is in **standard form** if it is written as $y = ax^2 = bx + c$.

(2) A quadratic is in **vertex form** if it is written as $y = a(x - h)^2 + k$, where (h, k) is the vertex of the parabola.

(3) A quadratic is in **factored form** if it is written as $y = a(x - r)(x - s)$, where $x = r$ and $x = s$ are its roots.